Lec 08

No. Date. July 14, 2026

The road to wisdom? well it's ~~a~~ plain & simple to express:
Err... & err... & err again but less, & less & less

piet Hein

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Practice:

$$\int (1+x^2)^{41}\,dx \qquad u = 1+x^2, \quad du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$$

$$= \int u^{41}\frac{du}{2} \Rightarrow \frac{1}{2}\left(\frac{u^{42}}{42}\right)+C = \boxed{\frac{u}{84}+C}$$

$$\int \sin(\pi t)\,dt \qquad u = \pi t, \quad du = \pi\,dt \Rightarrow \frac{du}{\pi} = dt$$

$$= \int \sin(u)\cdot\frac{du}{\pi} \Rightarrow \cancel{A} - \frac{\cos(u)}{\pi}+C \Rightarrow \boxed{-\frac{1}{\pi}\cos(\pi t)+C}$$

$$\int_0^{\pi/2} \sin(x)\sec^2(x)\,dx = \int_0^{\pi/2}\frac{\sin(x)}{\cos^2(x)}\,dx \qquad u = \cos(x), \quad du = -\sin(x)\,dx, \quad -du = \sin(x)\,dx$$

$$= \int_1^0 \frac{-du}{u^2} \qquad \begin{array}{c|c} x & u \\ \hline 0 & 1 \\ \frac{\pi}{2} & 0 \end{array}$$

we know: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$$= \cancel{(-1)}\int_1^0 \frac{du}{u^2} = \int_0^1 u^{-2}\,dx = \frac{u^{-2+1}}{-2+1}\Big|_0^1 = \frac{u^{-1}}{-1}\Big|_0^1 =$$

$$-\frac{1}{u}\Big|_0^1 = -\frac{1}{1}\cancel{\left(-\frac{1}{0}\right)}$$ no meaningful answer!

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$$\int_1^e \frac{\ln(x)}{x}dx = \int_1^e \frac{1}{x}\ln(x)dx \qquad u = \ln(x) \quad du = \frac{1}{x}$$

$$= \int_0^1 u\,du \qquad \begin{array}{c|c} x & u \\ 1 & 0 \\ e & 1 \end{array} \qquad = \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

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$$\int x\ln(x^4-1)dx$$

$$= \int x\ln((x-1)(x+1)(x^2+1))dx$$

we know $\ln(ab)$
$= \ln(a) + \ln(b)$

$$= \int x\left[\ln(x-1) + \ln(x+1) + \ln(x^2+1)\right]dx$$

$$= \int x\ln(x+1)dx + \int x\ln(x-1)dx + \int x\ln(x^2+1)dx$$

$v = x+1$
$dv = dx$
$\int v\ln(v)d$
$= \int v\ln(v)dv -$
$\int \ln(v)dv -$
$(v\ln(v) - v)$

$u = x-1$
$du = dx$
$\int (u+1)\ln(u)du$
$= \int u\ln(u)du +$
$\int \ln(u)du +$
$u\ln(u) - u$

$w = x^2+1$
$dw = 2x\,dx$
$\frac{dw}{2} = x\,dx$
$\frac{1}{2}\int \ln(w)dw =$
$\frac{1}{2}(w\ln(w) - w)$

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$$\int \tan^2(\theta)\sec^2(\theta)\,d\theta \qquad \begin{aligned} u &= \tan(\theta) \\ du &= \sec^2(\theta)\,d\theta \end{aligned}$$

$$= \int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3(\theta)}{3} + C$$

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$$\int \tan^3(\theta)\sec^3(\theta)\,d\theta \qquad \begin{aligned} u &= \sec(\theta) \\ du &= \sec(\theta)\tan(\theta)\,d\theta \end{aligned}$$

we know: $1 + \tan^2(\theta) = \sec^2(\theta)$

$$= \int [\underbrace{\underbrace{\tan(\theta)}_{\sec^2(\theta)-1}}_{u^2-1}]^2 u^2\,du = \int (u^2-1)u^2\,du = \int (u^4-u^2)\,du =$$

$$\frac{u^5}{5} - \frac{u^3}{3} + C = \frac{1}{5}\sec^5(\theta) + \frac{1}{3}\sec^2(\theta) + C$$

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$$\int \frac{e^x}{e^{2x}+1}\,dx \qquad \begin{aligned} u &= e^x \\ du &= e^x\,dx \end{aligned}$$

$$= \int \frac{e^x}{(e^x)^2+1}\,dx = \int \frac{1}{u^2+1}\,\cancel{dx}\,du = \text{antiderivative} \Rightarrow$$

$$\arctan(u) + C = \arctan(e^x) + C$$

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$$\int \frac{e^{2x}}{\sqrt{e^x+1}}\,dx = \int e^{2x}(e^x+1)^{-1/2} \qquad \begin{aligned} u &= e^x \\ du &= e^x\,dx \end{aligned}$$

$$= \int \frac{u}{\sqrt{u+1}}\,du \quad \begin{aligned} w &= u+1 \\ dw &= du \end{aligned} \quad = \int \frac{w-1}{\sqrt{w}}\,dw = \int \left(\frac{w}{\sqrt{w}} - \frac{1}{\sqrt{w}}\right)dw \Rightarrow$$

$$\int (w^{1/2} - w^{-1/2})\,dw = \frac{w^{3/2}}{3/2} - \frac{w^{1/2}}{1/2} + C = \frac{2}{3}w^{3/2} - 2w^{1/2} + C \Rightarrow$$

$$\frac{2}{3}(u+1)^{3/2} - 2(u+1)^{1/2} + C = \boxed{\frac{2}{3}(e^x+1)^{3/2} - 2(e^x+1)^{1/2} + C}$$

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Another way to solve it:

$$\int \frac{e^{2x}}{\sqrt{e^x+1}}\,dx \qquad t=\sqrt{e^x+1}=(e^x+1)^{\frac{1}{2}} \iff t^2=e^x+1 \iff e^x=t^2-1$$

$$dt=\frac{1}{2\sqrt{e^x+1}}\left(\frac{d}{dx}(e^x+1)\right)=\frac{e^x}{2\sqrt{e^x+1}}\,dx \Rightarrow$$

$$2dt=\frac{e^x}{\sqrt{e^x+1}}\,dx$$

$$=\int e^x\,2dt=\int (t^2-1)\,2dt=2\left[\frac{t^3}{3}-t\right]+C=$$

$$2\left[\frac{(e^x+1)^{3/2}}{3}-(e^x+1)^{\frac{1}{2}}\right]+C=$$

$$\boxed{\frac{2}{3}(e^x+1)^{3/2}-2(e^x+1)^{1/2}+C}$$

Integration by parts:

$$(u.v)'=u'.v+u.v'$$

$$u.v'=(u.v)'-u'.v$$

$$\int u.v'\,dx=u.v-\int u'.v\,dx$$

$$\int x\ln(x)\,dx \qquad u=\ln(x) \quad u'=\frac{1}{x} \qquad v=\frac{x^2}{2} \quad v'=x$$

$$=\frac{x^2}{2}\ln(x)-\int\frac{1}{x}\cdot\frac{x^2}{2}\,dx=\frac{x^2}{2}\ln(x)-\int\frac{x}{2}\,dx=$$

$$\frac{x^2}{2}\ln(x)-\frac{1}{2}\cdot\frac{x^2}{2}+C=\boxed{\frac{x^2}{2}\ln(x)-\frac{x^2}{4}+C}$$

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$\int x\cos(x)\,dx$ $\quad u=\cos(x) \quad \cancel{v'=x}$
$\quad u'=-\sin(x) \quad \cancel{v=\frac{x^2}{2}}$

$\cancel{= \frac{x^2}{2}\cos(x) - \int \frac{x^2}{2}(-\sin(x))\,dx}$

Instead do: $u=x \quad v'=\cos(x)$
$u'=1 \quad v=\sin(x)$ so $\Rightarrow$

$x\sin(x) - \int 1\cdot\sin(x)\,dx = x\sin(x) - (-\cos(x)) + C$

$= \boxed{x\sin(x) + \cos(x) + C}$

check:

$\frac{d}{dx}(x\sin(x) + \cos(x) + c) = \left[\frac{dx}{dx}\right]\sin(x) + \left[\frac{d}{dx}\sin(x)\right] - \sin(x) + 0$

$\Rightarrow \cancel{\sin(x)} + x\cos(x) - \cancel{\sin(x)} + 0 \Rightarrow \boxed{x\cos(x)}$

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$\int e^x\cos(x)\,dx$ $\quad u=\cos(x) \quad v'=e^x$
$\quad u'=-\sin(x) \quad v=e^x$

$= e^x\cos(x) - \int(-\sin(x))e^x\,dx = e^x\cos(x) + \int e^x\sin(x)\,dx$

$\cancel{s=e^x \quad t'=\sin(x)}$
$\cancel{s'=e^x \quad t=-\cos(x)}$

$\cancel{= e^x\cos(x) + e^x(-\cos(x)) - \int e^x(-\cos(x))\,dx}$

try instead: $s=\sin(x) \quad t'=e^x$
$s'=\cos(x) \quad t=e^x$

$= e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x)\,dx \Rightarrow$

$2\int e^x\cos(x)\,dx = e^x\cos(x) + e^x\sin(x) \Rightarrow \int e^x\cos(x)\,dx = \frac{e^x\cos(x) + e^x\sin(x)}{2}$

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$$\int \ln(x)\,dx \qquad u = \ln(x) \quad v' = 1 \qquad u' = \frac{1}{x} \quad v = x$$

$$= x\ln(x) - \frac{1}{x}x + C = x\ln(x) - \int 1 \cdot dx =$$

$$x\ln(x) - x + C$$

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$$\int x \arctan(x)\,dx \qquad u = \arctan(x) \quad v' = x \qquad u' = \frac{1}{x^2+1} \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2}\arctan(x) - \int \frac{x^2}{x^2+1} \cdot \frac{1}{2}\,dx \Rightarrow$$

$$\frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2}{x^2+1}\,dx \Rightarrow$$

$$\frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2+1}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx \Rightarrow$$

$$\frac{x^2}{2}\arctan(x) - \frac{1}{2}x + \frac{1}{2}\int \frac{1}{x^2+1}\,dx \Rightarrow$$

$$\boxed{\frac{x^2}{2}\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C}$$

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$$\int_0^1 x^2 e^x \, dx \qquad u = x^2 \quad v' = e^x$$
$$u' = 2x \quad v = e^x$$

$$= x^2 e^x - \int 2x e^x \, dx \qquad s = x \quad t' = e^x$$
$$s' = 1 \quad t = e^x$$

$$= x^2 e^x - 2\left[x e^x - \int 1 . e^x \, dx\right] \Rightarrow$$

$$\left(x^2 e^x - 2x e^x + 2e^x\right)\Big|_0^1 \Rightarrow 1^2 e^{(1)} - 2(1)e^{(1)} + 2e^{(1)} - (0 - 0 + 2e^0)$$

$$\Rightarrow \boxed{e - 2}$$

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whichever is first on the list goes into u.

logarithmic (including inverse hyperbolic)
Inverse trig functions
Polynomials (etc)
Trigonometric functions
Exponential (including hyperbolic)